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THE EFFECT OF FLEXIBLE PIPE NON-LINEAR BENDING STIFFNESS BEHAVIOR ON BEND STIFFENER ANALYSIS

Marcelo Caire
COPPE/Federal University of Rio de Janeiro

Murilo Augusto Vaz
COPPE/Federal University of Rio de Janeiro

ABSTRACT

Bend stiffeners are critical components for flexible risers and umbilical cables employed to ensure a safe transition at the riser-vessel interface, avoiding overbending and accumulation of high cyclic fatigue damage. The analysis and design of bend stiffeners usually consider the system as a unique beam, in which the pipe bending response is linear. However, the structural mechanics of these complex layered structures is governed by internal friction mechanisms that yield non-linear moment versus curvature relationship. In fact, the pipe structure exhibits an approximately bi-linear hysteretic bending moment against curvature relationship arising from the progressive activation of friction and consequential slipping between adjacent layers. The flexible pipe bending stiffness substantially reduces after a given critical curvature (i.e., after slip between adjacent layers) is reached. In this paper, the effect of this flexible pipe non-linear response on the bend stiffener design is evaluated. The mathematical formulation and the solution methodology are presented. A set of four non-linear ordinary differential equations is obtained from geometrical compatibility, equilibrium of forces and moments and constitutive equations and a numerical solution is obtained using the shooting method. A finite element analysis is developed to validate the analytical model and to assess the effect of the radial clearance between the structures on the bend stiffener response. A case study is presented for some static loading conditions and it is observed that the bending stiffness bi-linear behavior may not affect the bend stiffener extreme load design results, but it may significantly influence the fatigue analysis.

INTRODUCTION

Flexible pipes are complex composite structures made of various polymeric and metallic layers which play specific structural or functional roles. This multilayered structure yields high axial and torsional stiffness concomitantly ensuring very low bending stiffness. The pipe top connection is a critical point regarding the maximum allowed curvature and fatigue life because this region is susceptible to the highest stresses due to static and dynamic loads. The stiffness transition between the flexible pipe and the rigid platform is achieved using a conical shape structure made of polyurethane called a bend stiffener. In some cases the bend stiffener may be designed with an initial cylindrical section. This is an effective way to prevent the riser failure from overbending or from accumulation of fatigue damage. Besides, the stiffener must be designed to ensure its own fatigue lifetime.

Nowadays, the greatest part of the Brazilian oil and gas production is carried through flexible pipes. The growing demand for this product and its utilization in deeper waters have required better knowledge of the possible failure modes for the flexible pipe itself, end-fitting and bend stiffener. Recent failures with stiffeners at Campos Basin and the increasing number of FPSOs and monobuoys have driven research to better understand the flexible pipe/stiffener system response.

The analysis and design of bend stiffeners represented by an equivalent beam were previously presented. Vaz & Lemos [1] introduced the polyurethane material non-linearity and extended Boef & Out [2] model. Caire *et al* [3] presented a model considering the polyurethane with viscoelastic response which is an inherent characteristic to polymers.

In past analytical models the authors were concerned with the stiffener material influence on the system response. They considered the flexible pipe a homogeneous structure with linear elastic isotropic response and with no gap between the structures. It is however well known that the pipe constitutive response depends on a complex interaction mechanism between its layers. The structure exhibits a non-linear hysteretic bending moment x curvature response arising from the stick-slip mechanism due to friction between adjacent layers. At low curvatures there is no relative sliding between the layers. As the bending moment increases the wires slip and the bending stiffness significantly decreases. The usual design strategy is to consider a constant full-slip bend stiffness, which may lead to conservative results.

A question that arises when dealing with pipe bi-linear bending response is the effect of radial clearance between the stiffener and the pipe. With this gap consideration, some riser segments might overcome the critical curvature before it would without a gap. It may lead to a different curvature distribution alongside the pipe length regarding the usual response of a unique beam, which induces a change in the stiffener response.

In this paper, the effect of a flexible pipe non-linear bending response on the bend stiffener design is evaluated. An analytical mathematical formulation representing the pipe/stiffener system and the solution methodology are presented. Also, a finite element model is developed to validate the analytical model and to assess the effect of the gap between the structures on the system response. A case study is presented for some loading conditions and the results are discussed.

NOMENCLATURE

k	curvature
k_{cr}	critical curvature
$k_{\lim}$	limiting curvature ($1/MBR$)
EI_{ns}	no-slip pipe bending stiffness
EI_{fs}	full-slip pipe bending stiffness
EI_{BS}	stiffener bending stiffness
M	bending moment
T	axial force
V	shear force
I	second moment of area
F	force
ϕ , α	angle
x , y	coordinate
s	arc-length

BENDING BEHAVIOR OF FLEXIBLE PIPES

A non-linear hysteretic bending moment x curvature relationship characterizes the behavior of flexible pipes under bending, as shown in Figure 1. It depends on the geometrical and material properties of all layers and is greatly influenced by the interlayer contact pressure and friction coefficient. While the polymeric layers introduce material non-linearity, the helical layers may introduce geometrical non-linearity.

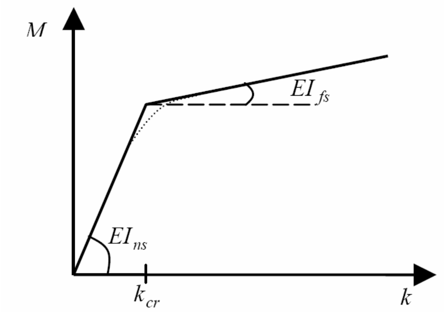

Figure 1 – Bending Moment x Curvature

As the internal friction impedes relative sliding between the layers the armour wires are “bound” to the pipe and must stretch when curved, hence the pipe exhibits high bending stiffness EI_{ns} . As the bending moment increases, relative slip begins after a critical value of curvature k_{cr} , the point at which the static friction is overcome and only cinematic friction will eventually remain after the full slippage. The bending stiffness parameter EI_{fs} significantly decreases. Although this non-linear process is actually smooth, this path may be modeled as a bi-linear stiffness with transition at the critical curvature.

An analytical expression to estimate the bending stiffness is basically derived from complex differential geometry considerations and has been performed by many authors: Feret & Bournazel [4], Out & Van Morgen [5], Witz & Tan [6], McIver [7], Kebadze & Kraincanic [8] and others. Although the analytical expressions present some slight differences from one author to another, they agree that the critical curvature is a function of the loading conditions, being highly affected by the resultant interlayer contact pressure.

Among the experimental work regarding evaluation of this parameter, it may be mentioned Vaz *et al.* [9], Fang & Lyons [10], Saevik & Berge [11], Witz [12], Magluta *et al* [13] and Troina *et al* [14]. Magluta *et al* [13] performed a set of static and dynamic experimental tests on a flexible riser specimen in order to study the influence of internal pressure and axial tension and they concluded that the bending stiffness is not affected by these loadings.

The usual stiffener design strategy is to consider a constant full slip pipe bend stiffness, but it is felt that this may lead to conservative results. The bending stiffness is not affected by the loading conditions but the critical curvature is. As a flexible riser is subjected to different values of axial loads, torsional moments and internal and external pressures along its length and through its operational conditions, the contact pressure between adjacent layers and hence the critical curvature also varies. A correct characterization of this parameter in the top connection is essential to develop a bi-linear model.

ANALYTICAL FORMULATION

The design of a bend stiffener for a given flexible riser and environmental conditions can be based on a slender beam model as shown in Figure 2.

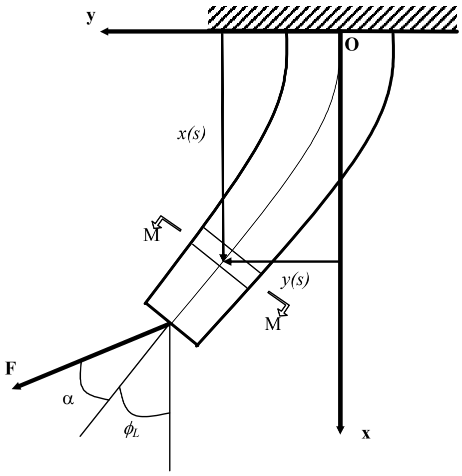

Figure 2 – Schematic of the pipe/stiffener system

The following simplifying assumptions are considered:

a) Cross-sections remain plane after deformation (Bernoulli-Euler formulation);
b) Cross-sections experience large displacements but strains are small;
c) Polyurethane is assumed linear elastic and symmetric;
d) Bending moment x curvature pipe bi-linear response;
e) Pipe and the stiffener are inextensible;
f) Gap between the structures is disregarded;
g) Stiffener is conical;
h) Critical curvature is assumed constant.

Applying trigonometrical relations to an infinitesimal element, the following relations are obtained,

$$\frac{dx}{ds} = \cos\phi(s) \tag{1}$$

$$\frac{dy}{ds} = \sin\phi(s) \tag{2}$$

where s is the beam arc-length ($0 \le s \le L$) , $\left[x(s), y(s)\right]$ are the Cartesian coordinates and $\phi(s)$ is the angle between the tangent and the x axis. Furthermore, the curvature $k(s)$ is given by,

$$\frac{d\phi}{ds} = k(s) \tag{3}$$

Considering static equilibrium, the force and moment reactions may be calculated in the origin of the Cartesian axis from the loading conditions (F, α, ϕ) . The tension $T(s)$, shear force $V(s)$ and bending moment $M(s)$ can be easily determined as follows,

$$V(s) = -F.\sin\left(\phi_L + \alpha - \phi(s)\right) \tag{4}$$

$$T(s) = F.\cos\left(\phi_L + \alpha - \phi(s)\right) \tag{5}$$

$$\begin{aligned} M(s) = {} & F\sin(\phi_L + \alpha)\int_s^L \cos\phi(s).ds \\ & - F\cos(\phi_L + \alpha)\int_s^L \sin\phi(s).ds \end{aligned} \tag{6}$$

Differentiating equation (6) with respect to s ,

$$\frac{dM(s)}{ds} = -F.\sin\left(\phi_L + \alpha - \phi(s)\right) \tag{7}$$

Using the assumptions (a-d), the following bending moment x curvature relation is applied to the system before and after the slip between adjacent layers,

$$M = \left(EI_{ns} + EI_{BS}(s)\right).k \qquad k \le k_{cr} \tag{8}$$

$$\begin{aligned} M = {} & \left(EI_{fs} + EI_{BS}(s)\right).k \\ & + (EI_{ns} - EI_{fs}).k_{cr} \qquad k > k_{cr} \end{aligned} \tag{9}$$

where EI_{ns} is the no-slip pipe bending stiffness, EI_{fs} is the full-slip pipe bending stiffness, EI_{BS} is the stiffener bending stiffness and k_{cr} is the critical curvature. Differentiating the constitutive equations (8) and (9) with respect to s and using (7) it is found,

$$\frac{\partial k}{\partial s} = -\frac{1}{EI_{ns} + EI_{BS}}\begin{pmatrix} \dfrac{\partial EI_{BS}}{\partial s}.k \\ +F\sin.(\phi_L + \alpha - \phi) \end{pmatrix} \qquad k \le k_{cr} \tag{10}$$

$$\frac{\partial k}{\partial s} = -\frac{1}{EI_{fs} + EI_{BS}}\begin{pmatrix} \dfrac{\partial EI_{BS}}{\partial s}.k \\ +F\sin.(\phi_L + \alpha - \phi) \end{pmatrix} \qquad k > k_{cr} \tag{11}$$

The geometrical relations (1) to (3) and equations (10) and (11) form the system of four non-linear ordinary differential equations that represents the boundary value problem. The following four boundary conditions are specified to solve the problem,

$$x(0) = y(0) = \phi(0) = \phi(L) - \phi_L = 0 \tag{12}$$

The numerical solution is obtained using the shooting method, which is based on converting the boundary-value problem in an equivalent initial value problem. A trial and error approach is then implemented to solve the problem. The mathematical package Mathematica v5.1 has been employed in the algorithm implementation.

FINITE ELEMENT MODEL

In order to compare and validate the analytical model, a finite element formulation with same assumptions is implemented using the software Abaqus v6.4. This correspondence is achieved as follows,

- two dimensional Euler-Bernoulli beam element B23 is used for the flexible pipe and bend stiffener;
- the *Step, Nlgeom option is used to consider the large displacement and material non-linearity;
- the stiffener tapered cross-section is considered using the command *Beam General Section;
- the pipe nonlinear bending moment x curvature relationship is included using the command *Beam General Section;
- a high value of axial stiffness of the pipe is given;
- the structures are tied together using the command *TIE.

As the gap between the pipe and the stiffener is not considered in the analytical formulation, a second finite element model is developed to evaluate its effect on the system response. For this model, ITT21 contact elements were used with a slide line defined with the *SLIDE LINE option. It assumes that the relative motion of the pipe/stiffener is predominantly along the line defined by the axis of one tube. The radial clearance between the pipe and the stiffener is assigned using the option *INTERFACE.

CASE STUDY

The example considered in the case study is a 3.2 m long flexible riser segment with an internal diameter of 4 in. The extreme loading condition specified for bend stiffener design is: maximum end rotation 45^o and maximum top force 250 kN. The bend stiffener and system geometry are shown in Figure 3.

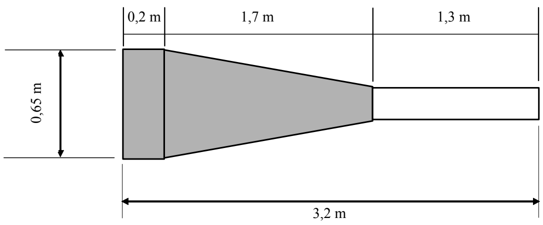

Figure 3 – System Geometry

The radial clearances considered in the finite element model with gap are 5 and 20 mm. To avoid numerical problems the pipe is extended with a segment of 0.2 m. In both finite element analyses the stiffener and pipe are respectively modeled with 190 and 320 beam elements, thus 190 contact elements were also necessary.

Two loadings are investigated (see Table 1) for extreme and typical fatigue life assessment.

Table 1 – Loading Conditions

Load Number	ϕ_L $[Degree]$	F $[kN]$
1 (Design)	45	250
2 (Fatigue)	5	250

The limiting curvature for the flexible riser is $k_{\lim} = 0.5\ m^{-1}$ ($MBR = 2.0\ m$). As the critical curvature varies considerably with the loading conditions imposed to the flexible riser (tension, internal and external pressures) a series of parametric values were used in the case study: 2.5, 5, 10, and 20% of the limiting curvature. The pipe full-slip bending stiffness is $EI_{fs} = 10\ kN.m^2$. In order to assess the bending stiffness influence before slip on the system response, three values were estimated: $EI_{ns} = 100, 1.000$ *and* $10.000\ kN.m^2$.

The polyurethane Young's modulus is assumed constant $E_{BS} = 45\,MPa$. It should be emphasized that the correct characterization of this value strongly influences the system response as observed by Vaz & Lemos [1].

RESULTS AND DISCUSSION

Design for extreme loading

In order to evaluate the effect of the pipe bi-linear bending response on the bend stiffener design the pipe curvature x arc-length is plotted in Figure 4 for the critical curvatures and for the highest value of no-slip pipe bending stiffness because it causes the largest influence on the system response. The bi-linear bending pipe behavior does not significantly change the curvature distribution within the bend stiffener region, except for the maximum value of critical curvature.

Figure 5 shows the influence of the no-slip pipe bending stiffness and critical curvature parameters on the maximum strain in the bend stiffener. The maximum strain occurs for the highest critical curvature ($k_{cr} = 20\%\,k_{\lim}$) and no-slip bending stiffness $EI_{ns} = 1000\,EI_{fs}$. The maximum strain in the bend stiffener increases only 1.8 % from the linear to the bi-linear analytical model, which is not very expressive.

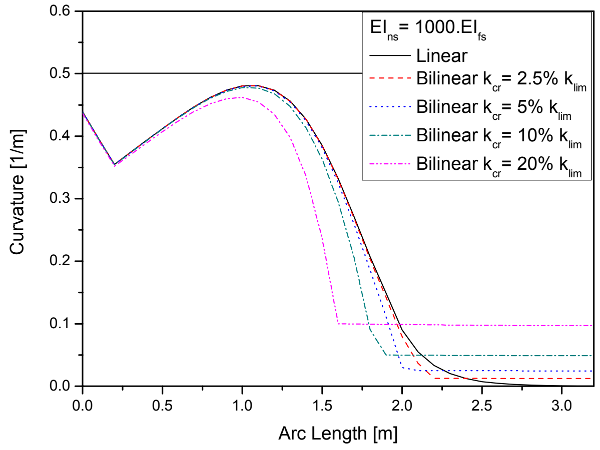

Figure 4 - Curvature x Arc-length
($F = 250\ kN, \phi = 45^o, EI_{ns} = 1000.EI_{fs}$)

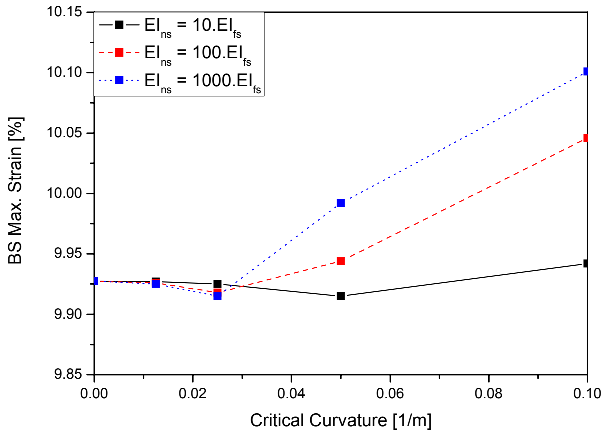

Figure 5 – BS Max. Strain x Critical Curvature
($F = 250\ kN, \phi = 45^o$)

The pipe/stiffener gap influence

Figure 6 shows results for the finite element model including gap between the stiffener and the pipe. It can be seen that the introduction of the gap leads to a decrease in the curvature distribution. The maximum curvature reduces approximately 4.2% from the linear to the bi-linear case and 5.9% from the bi-linear without and with gap. Hence a total drop of 10.2% is experienced from the linear to bi-linear with gap models for this extreme case. In addition if the gap is increased from 5 to 20 mm the curvature distribution is only slightly affected, thus showing that the gap should be included in respective of its magnitude as far as the extreme loading design conditions are concerned.

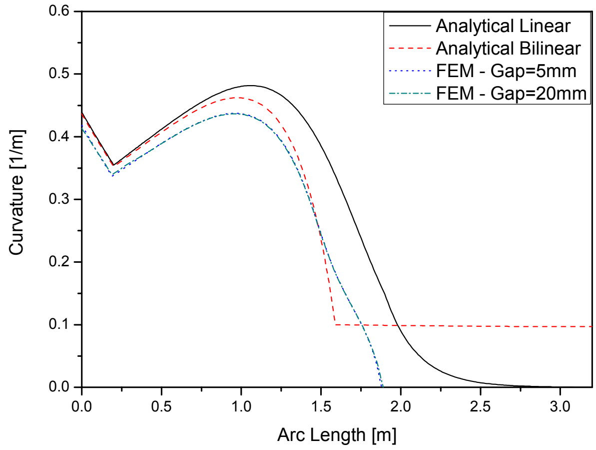

Figure 6 – Curvature x Arc-length
($F = 250\ kN, \phi = 45^o, EI_{ns} = 1000.EI_{fs}, k_{cr} = 20\% k_{\lim}$)

Design for fatigue life

To understand the possible effect that the pipe bi-linearity could cause on the fatigue analysis the results of curvature x arc-length are plotted in Figure 7 for the second loading condition, where the top end rotation is reduced to 5^o and the top force is maintained. The influence of the critical curvature on the curvature results is evaluated along the system length (s = 0, 1.0 and 2.0m) for a no-slip bending stiffness $EI_{ns} = 100\,EI_{fs}$. It is shown that the pipe bi-linearity reduces the curvature, especially for high values of critical curvature.

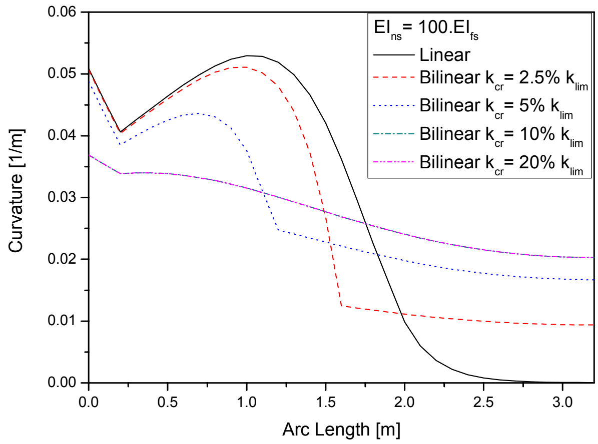

Figure 7 – Curvature x Arc-length
($F = 250\ kN, \phi = 5^o, EI_{ns} = 100.EI_{fs}$)

It is observed from Figure 8 that for s = 0 and 1.0m the increase of critical curvature decreases the system curvature, respectively 38 and 68%. At s = 2.0 m, i.e., 10 cm outside the bend stiffener, the curvature distribution increases with the critical curvature by approximately 144%. These values demonstrate the high influence of the bi-linearity on the curvature response.

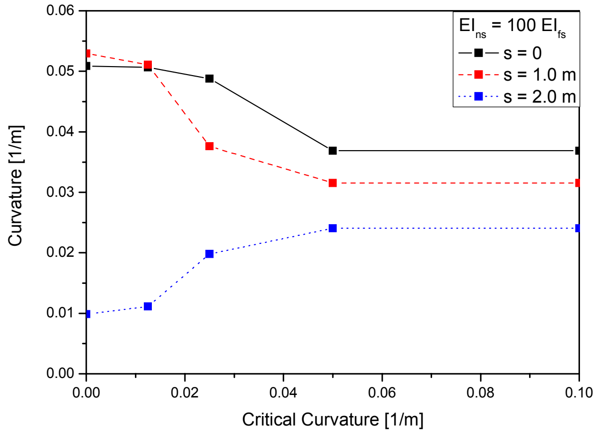

Figure 8 – Curvature x Critical Curvature
($F = 250\ kN, \phi = 5^o, EI_{ns} = 100.EI_{fs}$)

The pipe/stiffener gap influence on fatigue analysis

Figures 9 and 10 show that the introduction of a radial gap decreases the curvature at the built-in end. From Figure 9 ($k_{cr} = 2.5\% k_{\lim}$) it is observed that the maximum curvature increases 3% from the bi-linear analytical to the finite element model with a 20mm gap. For the highest value of critical curvature evaluated ($k_{cr} = 20\% k_{\lim}$), Figure 10 shows that the curvature increases 42% from the bi-linear analytical to the finite element model with a 20mm gap. It shows that when the critical curvature is increased from 2.5% to 20% of the limiting curvature the level of gap influence is greatest.

Regarding the effect of raising the gap on the fatigue analysis, it is observed in Figure 10 ($k_{cr} = 20\% k_{\lim}$) that as the gap is raised from 5 to 20mm the maximum curvature increases approximately 35%, while in Figure 9 ($k_{cr} = 2.5\% k_{\lim}$) 9%. It thus demonstrates the importance of the gap on the fatigue analysis.

Although only the bi-linear analytical model results with no gap were presented, the first finite element model with no gap was performed for each case study and the results with a high level of agreement support the analytical formulation for all cases.

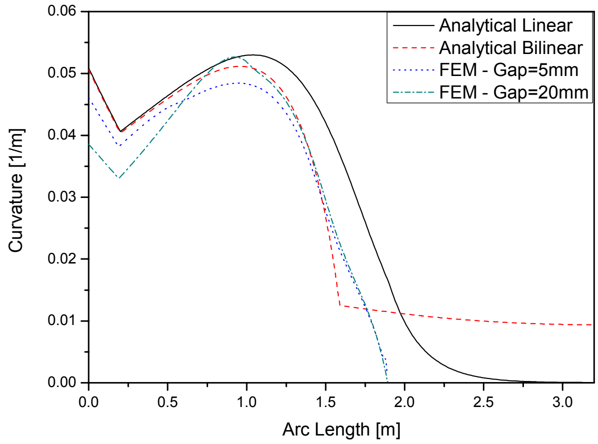

Figure 9 – Curvature x Arc-length
($F = 250\ kN, \phi = 5^o, EI_{ns} = 100.EI_{fs}, k_{cr} = 2.5\% k_{\lim}$)

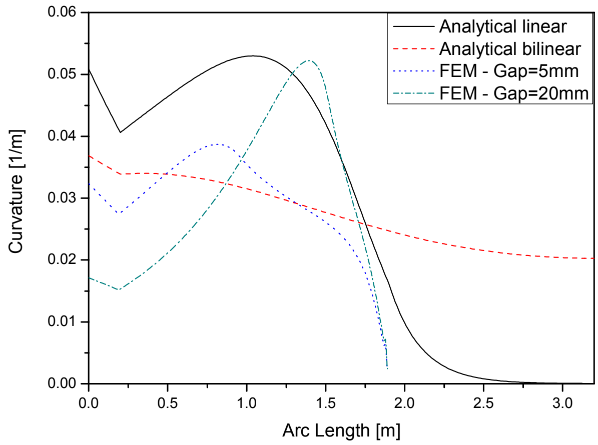

Figure 10 – Curvature x Arc-length
($F = 250\ kN, \phi = 5^o, EI_{ns} = 100.EI_{fs}, k_{cr} = 20\% k_{\lim}$)

CONCLUSIONS

In this paper the assumption of non-linear bending moment x curvature for the pipe is included in the beam model representing the flexible pipe/stiffener response. A finite element model and an analytical formulation are developed based on the same assumptions and a parametric analysis is performed for some case studies. Also, a second finite element model is developed to asses the effect of the gap between the structures. The main conclusions of this work are presented below:

- The bi-linear bending stiffness pipe behavior and the radial gap between the structures do not significantly affect the bend stiffener design if extreme loading conditions are evaluated. However these parameters may have some impact if fatigue loading is considered;
- If the bi-linear bending response and the gap are considered on design, it may lead to slightly less conservative results;
- The estimation of stresses and strains on the stiffener region for fatigue analysis should incorporate the effect of non-linear bending behavior and gap between the structures.

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